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Operational Value-at-Risk Software Library

Calculating the operational Value-at-Risk by using the actuarial approach

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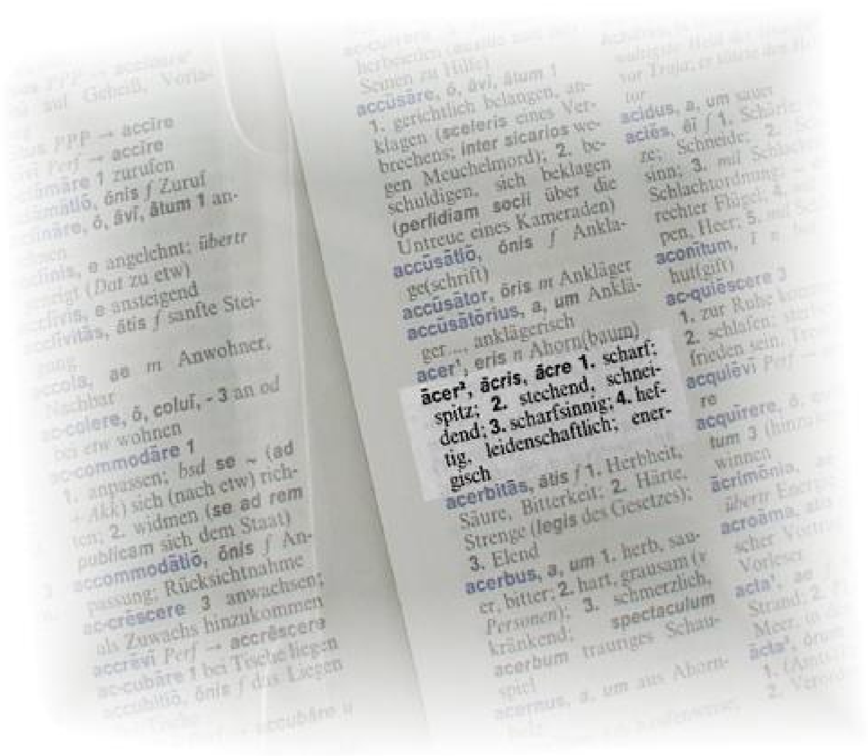

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Management Summary

The significance of operational risk management is rising. Spectacular loss events in the past (for example Barings, Daiwa, the german Metallgesellschaft, etc) underline the necessity for a proactive management of these risks. Through globalisation, progressing technologies or new business branches, new operational risks arise that need to be measured and quantified.

The regulatory authorities emphasize in the Basle II accord the rising importance of operational risks. These risks must be covered using equity capital. Basle II offers a wide range of mathematical tools to calculate the equity capital needed for operational risks. The methods differ by complexity and their 'hunger' for data needed for calculation.

Because of the fact, that operational risks need to be covered by equity capital not before 2007, many institutes sense themselves to be on the safe side because there are many years to come for implementing effective models to calculate the needed capital. Looking closer, one realises that terms like 'high impact, low frequency' emphasize the need to quickly start to collect data that is valid for calculation of operational value-at-risk. The more valid data the better. With only a few loss-values the calculation will end up in low-level statistics. Thus, no time to loose!

The following sections 'deriving parameters for VaR calculation from base data' and 'Calculating operational VaR using Monte-Carlo simulation' take a theoretical look at the mathematics one could use to calculate the operational value-at-risk using the actuarial approach. Subsequent to this follows a brief description of the Acrys Consult operational value-at-risk calculation software library. The last section takes a glance at a calculation example Acrys Consult uses to show the features of calculating the operational value-at-risk using the library.

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The actuarial approach

Overview

The actuarial approach was first used in the insurance business as a model to calculate potential losses. The use of this methodology to calculate the operational value-at-risk is a first step in developing a decent mathematical coverage of operational risk.

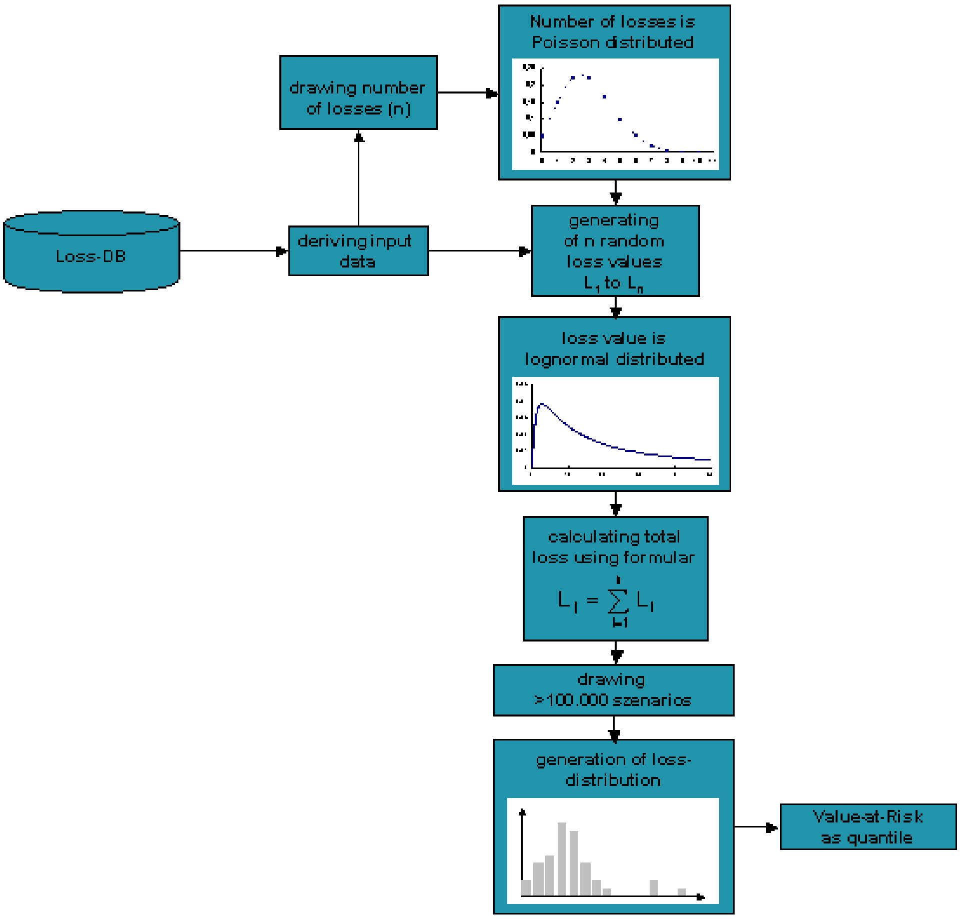

Figure 1: calculating operational Value-at-Risk

The above figure shows the complete process of calculating operational value-at-risk using the actuarial approach. Within the scope of the actuarial approach a loss is divided into two components: loss value and the number of losses representing the loss value. These components are stated to be independent and can therefore be modelled differently.

To model the number of losses, the Poisson-distribution is used. The loss values are usually modelled using the Lognormal-distribution. The possible loss events L_k are represented by the stochastical sum of all singular loss events L_k:

$$L_k = \sum_{i=0}^{N} L_i \quad \text{with } N \sim Po(\lambda) \text{ and } L_i \sim LogN(\mu; \sigma^2).$$

This context emphasizes again the independence of loss value L_i and the number of losses representing the loss value N. The distributions of loss value and the number of losses representing the loss value are defined through the average number of losses λ, the expected value of the losses μ and the standard deviation of the loss values σ. These parameters need to be derived from the base data.

The complete loss distribution is, as in this example, usually generated by a Monte-Carlo simulation wich draws a large number of scenarios using the above formular. The actual value-at-risk is then evaluated by a given quantile.

Deriving parameters for VaR-calculation from base data

Deriving n, μ, δ from base data

The following parameters need to be derived from the base data:

n is used as input parameter for the Poisson distribution function within Monte-Carlo simulation.

$\mu_{standard}$ and $\delta_{standard}$ are used to calculate $\mu_{lognorm}$ and $\delta_{lognorm}$ used by the Monte-Carlo simulation.

Using formulars (3-1) and (3-2) from the Appendix follows

$$\mu_{standard} = \text{mean(LogNormDistrib(} \mu_{standard} \text{ and } \delta_{standard} \text{))} \qquad (1\text{-}1)$$

$$\delta_{standard} = \text{variance(LogNormDistrib(} \mu_{standard} \text{ and } \delta_{standard} \text{))} \qquad (1\text{-}2)$$

solving (1-1) and (1-2) for $\mu_{standard}$ and $\delta_{standard}$ follows:

$$\mu_{standard} = e^{\mu_{log\,norm} + \frac{\delta_{log\,norm}{}^2}{2}} \qquad (1\text{-}3)$$

$$\delta_{standard} = e^{2\mu_{log\,norm} + \delta^2_{log\,norm}} (-1 + \varepsilon^2_{log\,norm}) \qquad (1\text{-}4)$$

solving (1-3) and (1-4) for $\mu_{lognorm}$ and $\delta_{lognorm}$ results in:

$$\mu_{log\,norm} = \frac{1}{2}(2\ln(\mu_{standard}) - \ln(\frac{\mu^2_{standard} + \delta_{standard}}{\mu^2_{standard}})) \qquad (1\text{-}5)$$

(is *only* defined for $\mu_{log\,norm} > 0$) and

$$\delta_{log\,norm} = \sqrt{\frac{\mu^2_{standard} + \delta_{standard}}{\mu^2_{standard}}} \qquad (1\text{-}6)$$

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Calculating confidence intervals for n, μ, δ

The confidence intervals are used to calculate boundaries for the calculated value-at-risk used to describe the quality of the calculated data. See section 'Example VaR calculation' for further details.

The calculation of the confidence intervals for $\mu_{\log norm}$ and $\delta_{\log norm}$ requires some basic statistic-mathematical knowledge:

- To evaluate the confidence interval for $\mu_{\log norm}$, one proposes that the mean of a distribution is student-t distributed.
- To evaluate the confidence interval for $\delta_{\log norm}$, one proposes that the variance of a distribution is chi-square distributed.

With the proposals above and formular (3-4) follows

$$\Delta_{\mu_{\log norm}} = \mu \pm t_{\frac{\alpha}{2},m} \cdot \delta_M \qquad (1\text{-}7)$$

and

$$\Delta_{\delta_{\log norm}} = \left\{ \frac{\delta^2 \cdot N}{\chi^2_{N-1,\alpha_1}} ; \frac{\delta^2 \cdot N}{\chi^2_{N-1,\alpha_2}} \right\} \qquad (1\text{-}8)$$

with

$$\alpha_1 = \frac{1+\alpha}{2} \qquad (1\text{-}9)$$

$$\alpha_2 = \frac{1-\alpha}{2} \qquad (1\text{-}10)$$

α respresents the quantile used to describe the reliability of the calculated boundaries $\Delta_{\mu_{\log norm}}$ and $\Delta_{\delta_{\log norm}}$.

$t_{\frac{\alpha}{2},m}$ is the quantile of the t distribution calculated as the inverse of the cumulative density function of the t-distribution defined by

$$f(t) = \frac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{1}{2}\right) \cdot \Gamma\left(\frac{n}{2}\right)} \cdot n^{-\frac{1}{2}} \cdot \left(1 + \frac{t^2}{n}\right)^{-\frac{n+1}{2}} \qquad (1\text{-}11)$$

χ^2_{N-1,α_1} and $\chi^2_{N-1,\alpha 2}$ are the quantiles of the χ^2 distribution calculated as the inverse of the cumulative density function

$$f(\chi) = \frac{e^{-\frac{x}{2}} \cdot x^{\frac{n-1}{2}-1}}{2^{\frac{n-1}{2}} \cdot \Gamma\left(\frac{n-1}{2}\right)} \qquad (1\text{-}12)$$

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Calculating operational VaR using Monte-Carlo simulation

How Monte-Carlo simulation works

During a Monte-Carlo simulation a large number of scenarios is generated (see section ‘The actuarial approach’) in order to evaluate the frequency of the results afterwards. Usually between 10^4 and 10^7 scenarios are generated. The most important part of a simulation model like this are the distribution-specific random number generators. Starting point for any random number generator is a random number generator which generates uniformly distributed random numbers. The generator used in this library is a so-called ‘Lehmer random number generator’. To transform these random numbers into random numbers according to a different probability density, specific transformation functions are used.

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Acrys Consult VaR calculation software library

Brief Overview

Calculation of the operational value-at-risk is provided by a platform independent library written in C developed by Acrys Consult.

The library contains a set of functions used to

- derive n, μ, δ from the input data delivered by any attached loss database or other datasource
- calculate the OR-VaR using the not-converged method
- calculate the OR-VaR using the converged method

To calculate the VaR using the not converged method the Monte-Carlo simulation is used to simulate loss scenarios by calculating L_k with formular

$$L_k = \sum_{i=0}^{N} L_i \quad \text{mit N} \sim \text{Po}(\lambda) \text{ und } L_i \sim \text{LogNorm}(\mu; \sigma^2) \qquad (2\text{-}1)$$

and a fixed number of scenarios to be drawn.

To calculate the VaR using the converged method, Monte-Carlo simulation is used to simulate at least two loss scenarios using formular (2-1) with increasing number of scenarios S until

$$| VAR_{S1} - VAR_{S2} | \leq \Delta_{VaR,\max} \qquad (2\text{-}2)$$

This may save some time if a large number of Monte-Carlo simulations are to be calculated.

In both cases the resulting distribution density from the simulation is used to evaluate the operational value-at-risk as the lowest quantile α of the potential losses.

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Example VaR Calculation

Used base data

To show the process of calculating the operational Value-at-Risk Acrys Consult generated an example base-data set divided into internal data, which would be collected within the institute, and external data that would be derived from sources outside the institute in order to complete the loss distributions by adding ‘low frequency, high impact’ events. The data usually collected within the institute is of ‘high frequency, low impact’ nature.

The number of loss events collected within the base data is very low, as the purpose of the example is just to show the calculation and the dependency of the quality of the calculated Value-at-Risk on the underlying data.

Internal and external data are described in the figures below.

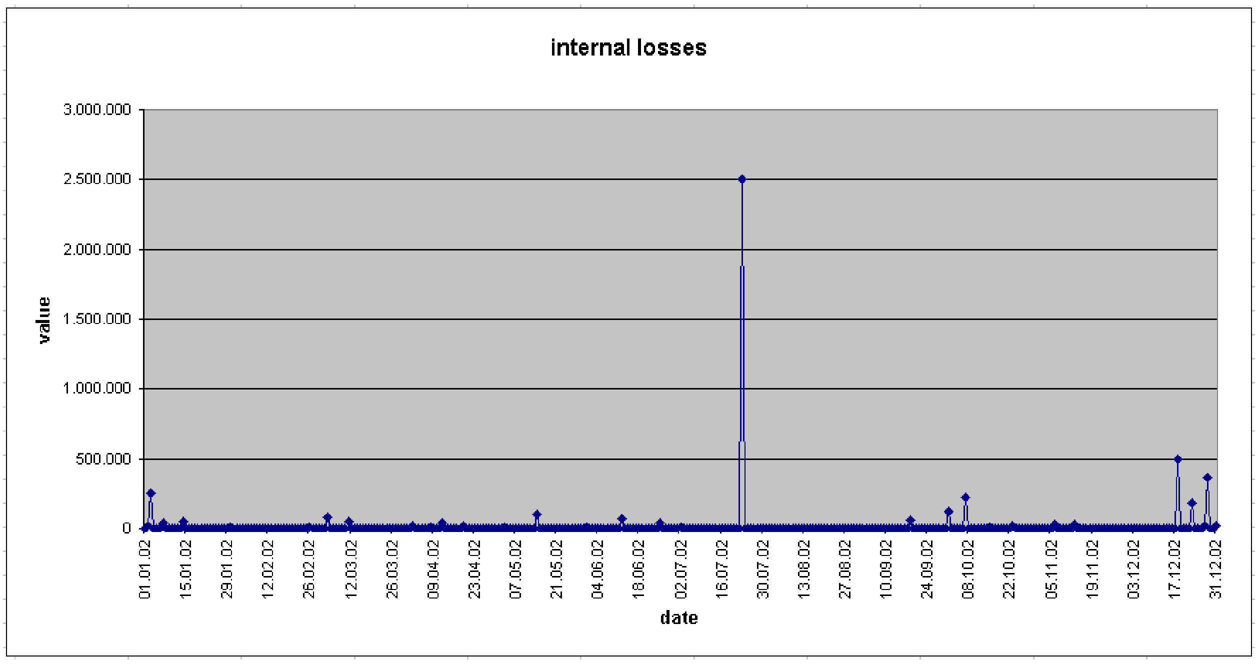

Figure 2: internal losses

The above internal losses example provides loss data starting at 01.10.2002 until 31.12.2002. It consists of ~ 360 singular loss events.

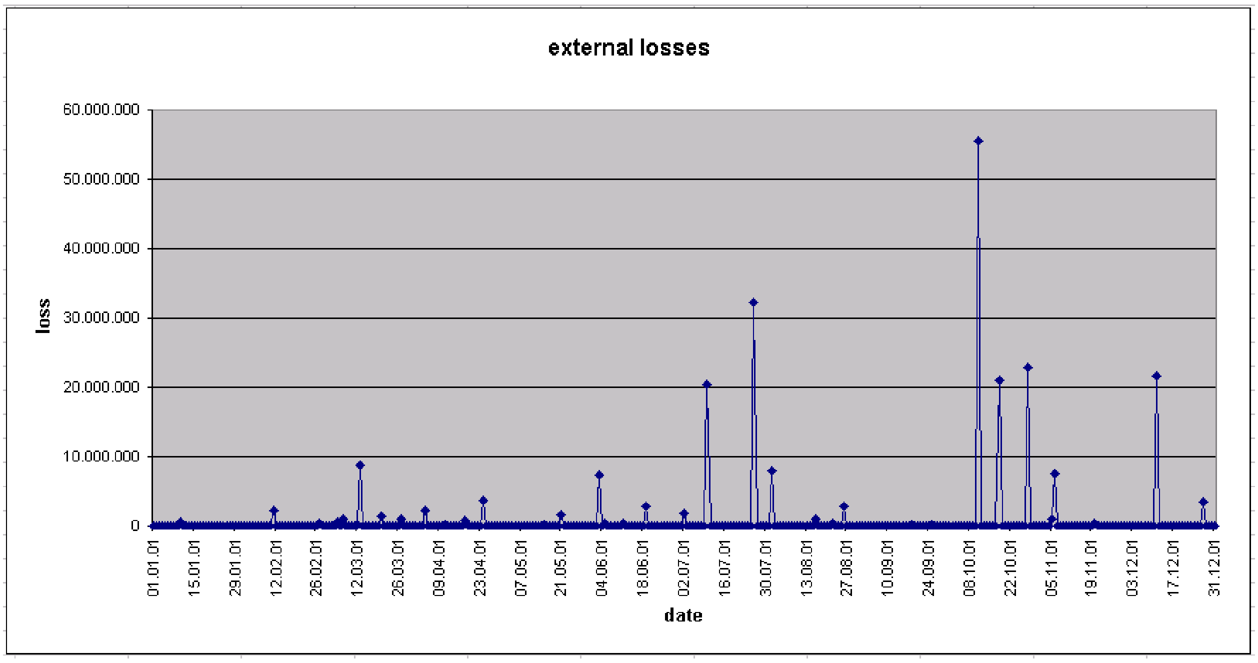

Figure 3: external losses

The above external losses example provides loss data starting at 01.10.2001 until 31.12.2001. It consists of ~ 45 singular loss events.

Calculating Value-at-Risk

At first, the Value-at-Risk is calculated using the internal data only. The result is shown by figure 4.

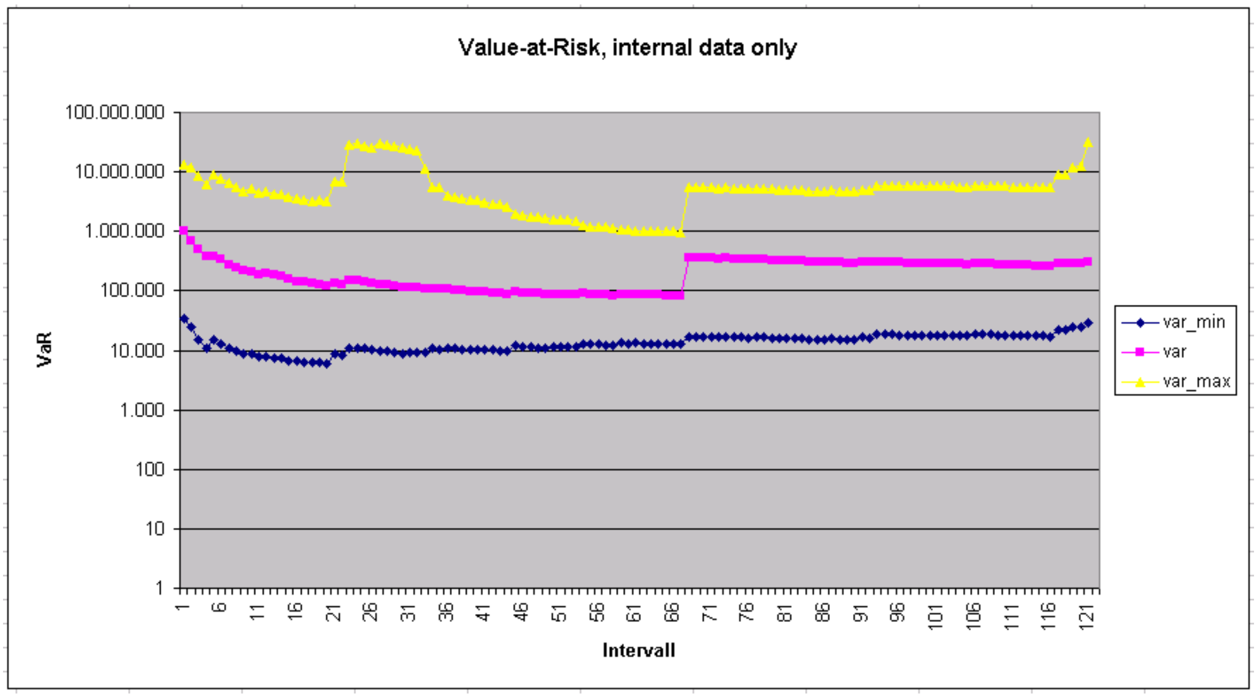

Figure 4: Value-at-Risk using internal data only

Var_min and var_max were calculated using the confidence intervals derived from the base data. One can easily state, that the band within which the calculated VaR lies is too wide to correctly measure the Value-at-Risk. One of the reasons for that is the fact, that there are too few datasets used for the calculation. If there were more datasets, the upper and the lower boundary of the VaR would converge towards the VaR, the reliability of the calculated VaR would increase.

This example shows the necessity to calculate not only the Value-at-Risk but also the upper and lower boundaries within which the defacto Value-at-Risk could dither.

As explained at the beginning of this section, external data is needed to complete the loss distribution. Figure 5 shows the calculation results for the internal data enriched with the external data.

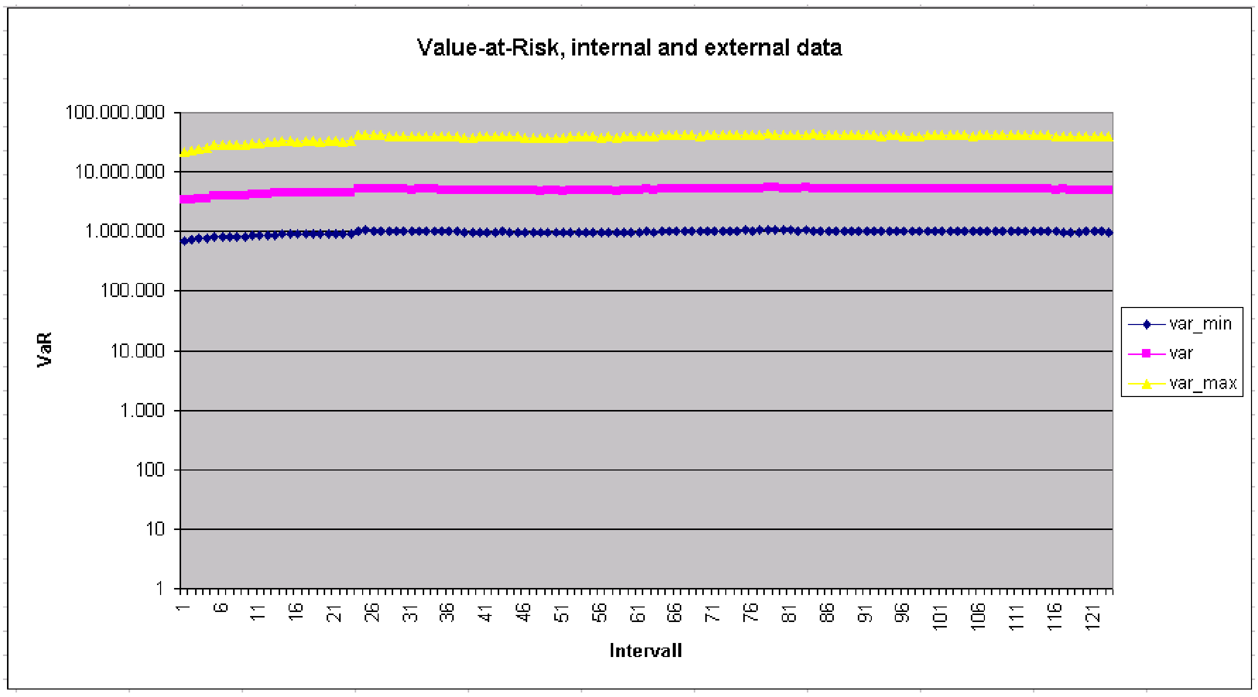

Figure 5: Value-at-Risk using internal and external data

The calculated Value-at-Risks are much ‘smoother’ as the VaR calculated using the internal data only. In conjunction with that, the boundaries var_max and var_min are closer together. This means that the reliability of the calculated VaR became better.

Appendix

Common statistical formulars used

Mean

$$\bar{x} = \mu = \frac{1}{N}\sum_{i=1}^{N} p_i \qquad (3\text{-}1)$$

Variance:

$$\delta^2 = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2 \qquad (3\text{-}2)$$

standard deviation

$$\delta = \sqrt{\text{var}} \qquad (3\text{-}3)$$

standard error (3-4)

$$\delta_M = \frac{\delta}{\sqrt{N}}$$

Our Services

Acrys Consult provides it's customers with sophisticated methodological and technological know-how regarding Operational Risks in general.

We recommend and support the modular tool suit *VÖB-ORC (Operational Risk Center)* of VÖB Bundesverband öffentlicher Banken Deutschlands and interexa AG with the following modules (please see our separate documentations in *www.acrys.com/services& competencies/ documentation):*

- ORC-RC: Risk Checkpoints (Self Assessment)
- ORC-RE: Loss Data Base
- ORC-RI: Risk Indicators
- ORC-RP: Risk Profile

Our operational VaR library is an easy to implement solution, technically independent. It can use any existing Loss Data Base.

Our operational VaR services offered are:

- Methodological and technological consulting
- Implementation of our OR VaR library

Contact

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